MESYS AG
Technoparkstrasse 1
CH–8005 Zürich
info@mesys.ch
T: +41 44 455 68 00

Mesys Shaft Analysis - Calculation Basis: DIN 743

Preface

The present document is intended to provide the users of MESYS Shaft Software with an overview of the calculation basis for the strength analysis of shafts and axles which is based on the German standard DIN 743 (not available in English). For further information on specific values, graphic data or guidelines, please refer to original document of the aforementioned standard when possible.

DIN 743: Calculation of load capacity for shafts and axles

Introduction

DIN 743 standard contains equations und methodical procedures for the verification of the load capacity of shafts and axles, as they are the responsible of many failure cases in mechanical engineering, in which the fatigue fracture is the main cause. The standard consists of four sections:

1- General (DIN 743-1)
2- Theoretical stress concentration factors and fatigue notch factors (DIN 743-2)
3- Strength of materials (DIN 743-3)
4- Fatigue limit, endurance limit – Equivalent damaging continuous stress (DIN 743-4)

The load capacity analysis is carried out by the determination of design safety factors against:

- Fatigue breakage (exceedance of fatigue limit)
- Static failure in consequence of maximum load damages (permanent deformations).

The calculated safety factor must be higher than the minimum required safety factor in order to validate a design.

Field of application

- Tensile/Compression, bending and torsion loads. Shear stresses due to shear loads are not taken into account.
- Temperature range: -40°C ≤ T ≤ 150°C
- The norm is only valid for non-welded steels.
- The environment cannot be corrosive.
- It is supposed that the amplitudes of the different loads will take place at same time (in the same phase).

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DIN 743-1: General

Evidence for avoidance of fatigue failure

Safety factor (S):

$$S = \frac{1}{\sqrt{\left(\frac{\sigma_{zda}}{\sigma_{zdADK}} + \frac{\sigma_{ba}}{\sigma_{bADK}}\right)^2 + \left(\frac{\tau_{ta}}{\tau_{tADK}}\right)^2}}$$

$$S \geq S_{min} = 1.2$$

$\sigma_{zda}, \sigma_{ba}, \sigma_{ta}$ Effective stress amplitudes due to tension/compression, bending, torsion.

$\sigma_{zdADK}, \sigma_{bADK}, \tau_{tADK}$ Permissible stress amplitudes.

Permissible amplitude. Fatigue limit of a piece part depending on the shape (σ_{zdADK}, σ_{bADK} τ_{tADK}):

The permissible stress amplitudes (σ_{zdADK}, σ_{bADK} and τ_{tADK}) are calculated from the fatigue strength of notched part (σ_{zdWK}, σ_{bWK} , τ_{tWK}) which in turn are calculated from the fatigue strength of a smooth test piece $\sigma_{zdW}(d_B)$, $\sigma_{bW}(d_B)$, $\tau_{tW}(d_B)$ according to a reference diameter, d_B:

$$\sigma_{zdWK} = \frac{\sigma_{zdW}(d_B) \cdot K_1(d_{eff})}{\mathrm{K}_\sigma}$$

$$\sigma_{bWK} = \frac{\sigma_{bW}(d_B) \cdot K_1(d_{eff})}{\mathrm{K}_\sigma}$$

$$\tau_{tWK} = \frac{\tau_{tWK}(d_B) \cdot K_1(d_{eff})}{\mathrm{K}_\tau}$$

where

$K_1(d_{eff})$ Technological size factor. Effect of heat treatment, depending on diameter (size of shaft) at the time of treatment.

$\sigma_{zdW}(d_B), \sigma_{bW}(d_B), \tau_{tWK}(d_B)$ Fatigue strength of a smooth test piece.

$K_\sigma = \left(\frac{\beta_\sigma}{K_2(d)} + \frac{1}{K_{F\sigma}} - 1\right) \cdot \frac{1}{K_V}$ General influential factor for tension/compression/bending.

$K_\tau = \left(\frac{\beta_\tau}{K_2(d)} + \frac{1}{K_{F\tau}} - 1\right) \cdot \frac{1}{K_V}$ General influential factor for torsion.

where

$K_2(d)$ Geometrical size coefficient: Effect of the decrease of bending strength against tensile strength as the diameter of the test piece increases.

$\beta_{\sigma,\tau}$ Fatigue notch factor for tension/compression, bending and torsion: Effect of the local stress concentrators.

$K_{F\sigma,\tau}$ Surface roughness factor for tension/compression, bending and torsion.

K_V Surface strain hardening factor: Effect of the compressive residual stresses

The standard considers two different cases:

CASE 1

It applies when the stress amplitude changes while the mean equivalent stress (σ_{mv} ; τ_{mv}) remains constant during a variation of the operational working load.

$$\sigma_{zdADK} = \sigma_{zdWK} - \Psi_{zd\sigma K} \cdot \sigma_{mv}$$

$$\sigma_{bADK} = \sigma_{bWK} - \Psi_{b\sigma K} \cdot \sigma_{mv}$$

$$\tau_{\tau ADK} = \tau_{\tau WK} - \Psi_{\tau K} \cdot \tau_{mv}$$

CASE 2

It applies under the assumption that the stress ratio between mean equivalent stress and effective stress amplitude (σ_{mv} /$\sigma_{zd,ba}$; τ_{mv} /τ_{ta}) remains constant during a variation of the operational working load.

$$\sigma_{zdADK} = \frac{\sigma_{zdWK}}{1 + \Psi_{zd\sigma K} \cdot \frac{\sigma_{mv}}{\sigma_{zda}}}$$

$$\sigma_{bADK} = \frac{\sigma_{bWK}}{1 + \Psi_{b\sigma K} \cdot \frac{\sigma_{mv}}{\sigma_{ba}}}$$

$$\tau_{tADK} = \frac{\tau_{tWK}}{1 + \Psi_{tK} \cdot \frac{\tau_{mv}}{\tau_{ta}}}$$

where

$$\sigma_{mv} = \sqrt{(\sigma_{zdm} + (\sigma_{bm})^2 + 3 \cdot \tau_{tm}{}^2}$$

$$\tau_{mv} = \frac{\sigma_{mv}}{\sqrt{3}}$$

and $\Psi_{zd,b\sigma K}$ and $\Psi_{\tau K}$ are the influencial factors for mean stress sensibility:

$$\Psi_{zd\sigma K} = \frac{\sigma_{zdWK}}{2 \cdot K_1\left(d_{eff}\right) \cdot \sigma_B(d_B) - \sigma_{zdWK}}$$

$$\Psi_{b\sigma K} = \frac{\sigma_{bWK}}{2 \cdot K_1\left(d_{eff}\right) \cdot \sigma_B(d_B) - \sigma_{bWK}}$$

$$\Psi_{\tau K} = \frac{\sigma_{\tau WK}}{2 \cdot K_1\left(d_{eff}\right) \cdot \sigma_B(d_B) - \sigma_{\tau WK}}$$

where

$\sigma_B(d_B)$ Ultimate tensile strength for the part at its reference diameter.

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Evidence for avoidance of permanent deformation, of incipient crack or overload failure

Permanent deformation

Safety factor (S):

$$S = \frac{1}{\sqrt{\left(\frac{\sigma_{zdmax}}{\sigma_{zdFK}} + \frac{\sigma_{bmax}}{\sigma_{bFK}}\right)^2 + \left(\frac{\tau_{tmax}}{\tau_{tFK}}\right)^2}}$$

$$S \geq \ S_{min} \ = 1.2$$

Where

$\sigma_{zdmax}, \sigma_{bmax}, \tau_{tmax}$ Existing maximal stresses due to the operational working loads for tension/compression, bending and torsion.

$$\sigma_{zd,FK} = K_1(d_{eff}) \cdot K_{2F} \cdot \gamma_F \cdot \sigma_s(d_B)$$

$$\sigma_{b,FK} = K_1(d_{eff}) \cdot K_{2F} \cdot \gamma_F \cdot \sigma_s(d_B)$$

$$\sigma_{zd,FK} = K_1(d_{eff}) \cdot K_{2F} \cdot \gamma_F \cdot \sigma_s(d_B)/\sqrt{3}$$

Where

K_{2F} Enlargement factor for static support effect as a result of local plastic deformations of the outer layer of the material (only for bending/torsion and non-hardened outer layer). No effect when tension/compression (K_{2F}=1).

γ_F Enlargement factor of the yield point by means of multi-axial stress state at notches and local strain hardening. In case of hardened outer layers or in absence of notches, γ_F=1.

$\sigma_S(d_B)$ Yield strength of a test piece for the reference diameter d_B; in case of hardened outer layers the values for the core are valid.

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Incipient crack or overload failure

Safety factor (S):

If compound stresses:

$$S=\frac{1}{0.5\cdot\left[\frac{\alpha_{\sigma zd}\cdot\sigma_{zdmax}}{\sigma_{zdBRand}}+\frac{\alpha_{\sigma b}\cdot\sigma_{bmax}}{\sigma_{bBRand}}+\sqrt{\left(\frac{\alpha_{\sigma zd}\cdot\sigma_{zdmax}}{\sigma_{zdBRand}}+\frac{\alpha_{\sigma b}\cdot\sigma_{bmax}}{\sigma_{bBRand}}\right)^2+\left(\frac{2\cdot\alpha_{\tau}\cdot\tau_{tmax}}{\tau_{tBRand}}\right)^2}\right]}$$

$S=\frac{\sigma_{bBRand}}{\sigma_{bmax}\cdot\alpha_{\sigma b}}$ (if bending only)

$S=\frac{\tau_{tBRand}}{\tau_{tmax}\cdot\alpha_{\tau}}$ (if torsion only)

Where

$\alpha_{\sigma zd}, \alpha_{\sigma b}, \alpha_{\tau}$	Stress concentration factors for tension/compression, bending and torsion.
$\sigma_{zdBRand}$	Ultimate tensile strength in the hardened outer layers during tension/compression, bending and torsion

This analysis is valid for hardened outer layers and heat-treated steels with σ_B > 1300 N/mm^2 and local extensibility under 4%. In case of brittle materials: $\sigma_{zdBRand}=\sigma_{BRand}$, $\tau_{tBRand}=\tau_{BRand}$ (please refer to the standard for specific values)

DIN 743-2: Theoretical stress concentration factors and fatigue notch factors

In this section of the norm, standardized graphics, formulas and values for both stress concentration factors and fatigue notch are available.

Technological size factors

According to the norm, the size influence must be taken into account with the three factors $K_1(d_{eff}), K_2(d), K_3(d)$ when d>7.5 mm depending on the diameter of the test piece:

Technological size factor, $K_1(d_{eff})$:

The technological size factor considers by approximation, that the reachable hardnesses during heat treatments (and thus also the yield point and fatigue strength), and correspondingly the core hardnesses during case hardening, diminish as the diameter increases. This coefficient does not depend upon the type of load (tension/compression, bending, and shear) and it is estimated with the d_{eff} diameter used for the heat treatment. $K_1(d_{eff})$ has to be considered in case that the strength of the part is not measured but rather calculated from the strength of the test piece as indicated in the standard.

For nitrided steel and ultimate tensile strength of both general and high strength steels as well as structural steel in non-heat-treated state, it must be used:		
$d_{eff} \leq 100$ mm	100 mm $\leq d_{eff} \leq$ 300 mm	300 mm $d_{eff} \leq$ 500 mm
$K_1(d_{eff}) = 1$	$K_1(d_{eff}) = 1 - 0.23 \cdot \log\left(\frac{d_{eff}}{100\ mm}\right)$	$K_1(d_{eff}) = 0.89$
The yield point for general and high strength steels as well as for other structural steels in non-heat-treated state has to be reduced by:		
$d_{eff} \leq 32$ mm	100 mm $\leq d_{eff} \leq$ 300 mm, d_B=16 mm	300 mm $d_{eff} \leq$ 500 mm
$K_1(d_{eff}) = 1$	$K_1(d_{eff}) = 1 - 0.26 \cdot \log\left(\frac{d_{eff}}{2 \cdot d_B}\right)$	$K_1(d_{eff}) = 0.75$
For Cr-Ni-Mo case-hardened steels in blank or case-hardened state and for the ultimate tensile strength of heat-treated steels as well as other structural steels in heat-treated state, it must be used:		
$d_{eff} \leq 16$ mm	16 mm $\leq d_{eff} \leq$ 300 mm, d_B=16 mm	300 mm $d_{eff} \leq$ 500 mm
$K_1(d_{eff}) = 1$	$K_1(d_{eff}) = 1 - 0.26 \cdot \log\left(\frac{d_{eff}}{d_B}\right)$	$K_1(d_{eff}) = 0.67$
For steels in blank or case-hardened state (except Cr-Ni-Mo), it applies:		
$d_{eff} \leq 16$ mm	16 mm $\leq d_{eff} \leq$ 150 mm, d_B=16 mm	150 mm $d_{eff} \leq$ 500 mm
$K_1(d_{eff}) = 1$	$K_1(d_{eff}) = 1 - 0.41 \cdot \log\left(\frac{d_{eff}}{d_B}\right)$	$K_1(d_{eff}) = 0.60$
For the yield point of heat-treated steels as well as other structural steels in heat-treated state, it has to be used:		
$d_{eff} \leq 16$ mm	16 mm $\leq d_{eff} \leq$ 300 mm, d_B=16 mm	300 mm $d_{eff} \leq$ 500 mm
$K_1(d_{eff}) = 1$	$K_1(d_{eff}) = 1 - 0.34 \cdot \log\left(\frac{d_{eff}}{d_B}\right)$	$K_1(d_{eff}) = 0.57$

For bigger diameters an extrapolation of the given values with the steel manufacturer should be agreed.

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Technological size factor, $K_2(d)$:

The geometrical size factor considers that, as the diameters or thicknesses increase, the alternating bending strength converge towards the tension/compression alternating strength, and that the torsion alternating strength will correspondingly decrease.

For tension/compression, it must be used:

d (arbitrary): $K_2(d) = 1$

For bending/torsion, it must be used:

16 mm ≤ d_{eff} ≤ 150 mm: $K_2(d) = 1 - 0.2 \cdot \frac{\log(d/7.5mm)}{\log 20}$

d_{eff} ≥ 150 mm: $K_2(d) = 0.8$

Technological size factor, $K_3(d)$:

This factor is the same as the factor $K_2(d)$ but for notched parts. This geometrical size factor considers the change of the fatigue notch factor, if the measurements of construction parts differ from the test piece messurements, and all measurements in the same scale have been changed. It will only be considered if fatigue notch factor have been defined experimentally and the reference diameter differs from the actual diameter of the construction part.

It depends of the stress concentration factors and is calculated as follows:

7.5 mm ≤ *d* ≤ 150 mm: $K_2(d) = 1 - 0.2 \log \alpha_\sigma \cdot \frac{\log(d/7.5mm)}{\log 20}$

d ≥ 150 mm: $K_2(d) = 1 - 0.2 \log \alpha_\sigma$ (α_τ if torsion)

Surface roughness factor, $K_{F\sigma,\tau}$:

It considers the additional influence of the roughness at the local stresses and thus at the endurance limit of the construction part. $K_{F\sigma}$ can be calculated for tension/compression or bending as follows:

$$K_{F\sigma} = 1 - 0.22 \log\left(\frac{R_Z}{\mu m}\right) \cdot \left(\log\left(\frac{\sigma_B(d)}{20\ N/mm^2}\right) - 1\right)$$

$$\sigma_B \leq 2000\ N/mm^2$$

Rz average roughness in µm (If a peak value of roughness larger than 2Rz appear in the notch, that value must be used instead of Rz)

For torsion, it will be used:

$$K_{F\sigma} = 0.575 K_{F\sigma} + 0.425$$

DIN 743-3: Strength of materials

The characteristic values apply for material samples with the d_B diameter and they are tabulated in the standard annexes. The characteristic values of ultimate tensile strength σ_B correspond to the lower bounds of the valid ranges of the small basic sizes (reference diameter d_B) that are indicated in the standards. With regard to the endurance limit, unless otherwise specified, the fatigue strength can be calculated as follows:

$\sigma_{bW} = 0.5 \cdot \sigma_B$ valid for $d_B \leq 7.5$ mm

$\sigma_{zdW} = 0.4 \cdot \sigma_B$ valid for $d_B \leq 7.5$ mm

$\tau_W = 0.3 \cdot \sigma_B$ valid for $d_B \leq 7.5$ mm

σ_B ($\sigma_B = R_m$) is valid for a temperature $\vartheta = 20$°C.

DIN 743-4: Fatigue limit, endurance limit – Equivalently damaging continuous stress

The method contained in this norm takes into account the whole load spectrum with number of load cycles under the break point N_D of the Wöhler curve. The fatigue strength drop as a consequence of big effective operational loads is determined by approximation. The order of influence in not considered. The assumptions for the Wöhler line are valid for notched circular rods. Please refer to the standard for further information on the calculation approach.